2023_“ShuWei Cup”

Problem D：The Mathematics of Laundry Cleaning

Laundry cleaning is something people do every day. The decontamination function of laundry detergent comes from some surfactant chemicals. They can increase water's permeability and utilize the intermolecular electrostatic repulsion mechanism to remove dirt particles. Due to the form of the surfactant molecules, laundry detergent can exert its dual function. One end of the surfactant molecule is lipophilic, which attracts dirt and repels water, while the other end is hydrophilic, attracting water molecules. When the laundry detergent is poured into water, the lipophilic part of the surfactant molecule vigorously adheres to any surface not filled with water, such as the object's surface being washed (due to the body or fabric surface). At the same time, the hydrophilic part repels oily substances. It weakens the intermolecular forces that maintain the binding of water molecules (the forces that cause water to form droplets like being wrapped in an elastic film), allowing individual molecules to penetrate between the surface of the object to be cleaned and the dirt particles. Therefore, it can be said that surfactants reduce the surface tension of water. The mechanical action of a washing machine or hand rubbing can cause the dirt particles surrounded by surfactant molecules on the surface to become dislodged, and the dirt particles adhere to the lipophilic portion of the surfactant

molecules. This causes the dirt particles still suspended on the object's surface to be removed during the Rinsing phase.

In practice, from small-scale household operations to professional operations in hotels and specialized institutions, the question that needs to be considered is: How can we get the laundry clean and tidy at a lower cost? This seemingly simple life problem contains profound mathematical principles. Please develop an appropriate mathematical model to solve the following problem:

1. A garment with dirt attached, if the amount of dirt and the amount of water available are given, and the solubility of the dirt in water at the k th wash is a_k, where $a_1 = 0.80, a_k = 0.5a_{k-1}, k = 2,3,\mathrm{L}$. Without considering other factors, what is the best way to clean it? Give the optimal solution regarding the number of washings and the amount of water used each time, and discuss the effect of a_k, the initial amount of dirt, and the amount of water available on the objective.

2. Assuming each washing takes the same time and there is no limit on available water, the final dirt residue should be no more than one-thousandth of the initial dirt amount, with other conditions similar to problem 1. Provide the most time-efficient cleaning plan and analyze the impact of a_k and the initial dirt amount on the optimal solution.

3. There are several pieces of clothing on which the type and quantity of dirt are shown in Table 1. The existing ten kinds of detergents,

the solubility of various detergents on the dirt, and detergent unit prices are shown in Table 2. If the water cost is 3.8 yuan per ton, try to give both cost savings and a good cleaning program.

4. There are several types of clothing with different materials, and the type and quantity of dirt on each are shown in Table 3. Considering differences in material, color, and other aspects, some clothes cannot be mixed for washing, as shown in Table 4. Under the same conditions as problem 2, attempt to provide a cost-effective and efficient cleaning plan.

Note: This competition only considers conventional water-washing methods.

Definition of terms:

1.The solubility a_k : represents the proportion of dirt dissolved by an equal amount of detergent during the k th washing relative to the initial dirt amount.

2. The initial dirt amount: represents the quantity of dirt on the clothing before cleaning, measured in grams.